Alphabetical Arrangement - Solution

In a mnemonic of length ℓ, there are a total of $\ell - 1$ two-letter combinations. For the first n letters of the alphabet, our mnemonic has to contain all of the n^2 different two-letter combinations at least once, so that $\ell \geq n^2 + 1$. Thus, the minimal length is $n^2 + 1$ and it turns out that we can in fact always find a mnemonic of length $n^2 + 1$ that works. For this, there are multiple strategies that work. We use the following construction, where we show inductively that for any n we can find a mnemonic of length $n^2 + 1$ that ends in an A. For $n = 1$ the mnemonic AA works. Suppose that we have a construction that works for $n = k$. With the numbers corresponding to the related letters, we add the following to the end of the construction for $n = k$:

$$(k+1), 2, (k+1), 3, (k+1), 4 \ldots, (k+1), k, (k+1), (k+1), 1.$$

As an example, our first three mnemonics are AA, AABBA and AABBACBCCA. It is easily verified that this addition contains all two-letter combinations with $k + 1$, except for $1, (k+1)$. However, by our hypothesis the construction for $n = k$ ends in an A, so that we also have this two-letter combination. Furthermore, any two letter combination without $(k+1)$ was already contained in the mnemonic for $n = k$. It is easy to verify these mnemonics are indeed of length $n^2 + 1$.